Mechanism Design - VCG

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1 Introduction

Topics discussed in this lecture are covered in [2], chapter 7, and [1], chapter 9 and beginning of chapter 11. Students are advised to consult that reference.

The notes here were made for my own use, to remind me during the lecture what to cover, and to remind me in the future what was covered.

2 Introduction

A is a set of alternatives.

$v_i : A \longrightarrow R$ (preference) valuation function for player i.

Social choice function $f : V_1 \times \ldots V_n \longrightarrow A$.

For example: social choice function may be an allocation of items, an optimal flow in a routing game, an outcome of an election.

The questions that we study are:

Axiomatic approach. Listing desirable properties of social choice functions, are they consistent (is there one function with all properties)? E.g.: every agent gets her MMS; for every candidate, the the winner of the election is preferred over that candidate by the majoriy of the voters (**Condorcet Paradox**).

Implementation. Given a consistent social choice function, how can we implemented it when players have private valuation functions.

We distinguish between various implementations.

DSIC: Implementation in dominant strategies: every player has a dominant strategy and if it is followed, the social choice function is implemented.

BNIC: Implementation in Bayesian-Nash equilibrium.

Mechanisms with or without money.

With money: assume **quasilinear utilities** – linear dependency on money.

What purposes can money serve? We can give incentives to agents through money. It allows to spread the social good among all participants, making it their interest to reach a common goal. Transfer utilities among players. Allows coupling of current and future economic transactions.

Example:

Allocate one item to one of several players.

Each has value. Values are comparable (measured in same units - money). Social welfare function – sum of received values.

Mechanism with money: Vickrey's second price auction.

Ignore payments, just an allocation problem. Money transfers do not affect the social welfare function.

3 VCG Vickrey, Clarke, Groves

A **direct revelation** mechanism with money.

A set of alternatives.

$v_i : A \longrightarrow R$ (preference) valuation function for player i.

Social welfare of alternative a: $\sum_i v_i(a)$.

Social choice function $f : V_1 \times \ldots V_n \longrightarrow A$. Here consider maximizing welfare.

Payment functions $p_i : V_1 \times \ldots V_n \longrightarrow R$.

Incentive compatible (implementation in dominant strategies by direct revelation).

For every player i, for all v_{-i} and v_i, v'_i, let a and a' denote $f(v_i, v_{-i})$ and $f(v'_i, v_{-i})$, and let ρ_i and ρ'_i denote $p_i(v_i, v_{-i})$ and $p_i(v'_i, v_{-i})$. Then $v_i(a) - \rho_i \geq v_i(a') - \rho'_i$.

Basic VCG mechanism. f maximizes social welfare. p pays the player the value that other players get.

Why is it dominant strategy to be truthful? Regardless of what others report, you can assume that their reports are truthful. Now your goal is for the mechanism to find the true optimal allocation with respect to the other reports and your own valuation function. If truthful, this will indeed happen.

General VCG mechanism. Okay to shift payment of player i by any amount that depends only on v_{-i}.

Multi item allocation.

Ask each player to report valuation function.

Idea - pay each player the value obtained by the other players. All have mutual interest of optimizing welfare.

Additional desirable properties for payment:

No positive transfers. Players should pay money, not get money.

Individual rationality. Players get nonnegative utility from participating.

(Think of an external game in which players decide whether to join the allocation game. We want the payment aspect to be such that players interested in the items will not fear to join, whereas players not interested will not join just for the hope of getting a payment.)

Clarke pivot rule: the shift in payment for agent i is by the maximum welfare that other players could have got without i. Default for VCG (unless stated otherwise).

Properties of VCG.

Money is a tool to implement a desirable social choice. Not intended to generate *revenue*. If n identical items and n players each of capacity 1, no one pays anything with Clarke pivot rule.

Envy freeness. No player looks at the plate of other players. For second price auction, the eventual solution involves no envy - no player wants for himself the outcome of a different player. More generally, this is not true. Two items and two players. One player has values 3 with capacity of 1, the other has value 2 with capacity of 2. Social welfare would give one item to each, put only first player would pay. Would like to switch with other player.

4 Characterizations of incentive compatible mechanisms

v_i: **type** of agent i (his valuation function for the alternatives).

Revelation principle. May as well consider direct revelation mechanisms in which every agent reveals his type. Can simulate any other mechanism. However, other mechanisms might be more efficient in terms of computation or communication requirements.

A mechanism is incentive compatible iff for every agent i and reports v_{-i} of other players:

1. The alternative $a = f(v_i, v_{-i})$ may depend on v_i, but the payment p_i depends only on a and on v_{-i} (if two v_i lead to the same a the payments are the same).

2. The alternative $a = f(v_i, v_{-i})$ is among those a that maximize $v_i(a) - p_i(a, v_{-i})$.

Much more is known regarding properties of IC mechanisms, depending on the domain of f. In some cases only VCG like mechanisms are IC, and in other cases there are other mechanisms.

5 GSP Generalized Second Price auction

k items (advertising slots). n bidders, each of capacity 1. Basic version. (Assuming *pay for impression*. For *pay per click*, scale bid by estimate of probability of click.)

VCG. Let each player provide a bid for each item. Find optimal allocation. Charge Clarke payments. If all items identical, pay $k+1$ highest bid. Problem: revenue might be too small. (Other issues include interdependence among bidders: I might be willing to pay so as not to have my competitor place an add. Not addressed here.)

GSP. Assume that items are ordered (from top to bottom), the order in which typical users scan a page. Allocated highest item to highest bid and so on. Each winner pays next bid.

Not truthful. However, higher revenues in practice. Also, easier to explain (initially, marketed as based on Nobel winning mechanisms), easier to submit bids.

Much more stable than previous version in which each winner paid his own bid.

6 Combinatorial auctions

m indivisible items and n bidders.

Type (valuation function) of a bidder: a nonnegative monotone normalized set function.

Social welfare of allocation: $\sum_{i=1}^{n} v_i(S_i)$.

VCG: pay the loss in welfare that you caused.

Complexity: 2^m in reporting v. Not practical for large m.

Single minded. Each player i has value $v_i > 0$ only for a specific set S_i.

Now reporting is easy. However, computing maximum welfare is NP-hard.

Reduction from Maximum Independent set, which is NP-hard to approximate wihin a ratio of $n^{1-\epsilon}$. Can be viewed as an allocation instance in which the players are the vertices, the items are the edges, and each agent is single minded (wants all edges incident with its vertex).

Best approximation ratio one can hope for is roughly $\sqrt{m}$, even in the unweighted case.

References

[1] Noam Nisan, Tim Roughgarden, Eva Tardos and Vijay V. Vazirani (Editors), Algorithmic Game Theory, Cambridge University Press, 2007. (Chapters 9 and 11.)

[2] Tim Roughgarden, Twenty Lectures on Algorithmic Game Theory, Cambridge University Press, 2016. (Mostly Chapter 7.)