Final assignment - AGT

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1 Introduction

You are welcome to consult references, but do this assignment on your own, and do not discuss it with anyone else. (In other words, if you still want to discuss topics of the course with other students, then put this assignment aside and do not read it yet. Read it only after you are ready to do the assignment on your own.)

You are welcome to use and adapt concepts learned in class. You may use without proof any theorem stated in class (or in homework assignments), if you state it properly. If you wish to use theorems not stated in class but stated elsewhere, then a proof is needed.

The final grade will depend on the contents of your answer, and also on how clear and well organized your answer is. Please hand in by July 31. If you need an extension, let me know.

If anything is not clear, do not hesitate to ask.

2 The final assignment

There are n equally entitled students and an apartment with n rooms. The total rent for the apartment is T. The students want to jointly rent the apartment, give each student one room, and jointly pay the rent. Rooms may have different characteristics (different sizes, windows facing different directions, may or may not include a private bathroom, etc.) Each student i has a nonnegative valuation function v_i over the rooms. The goal of this assignment is to explore mechanisms by which students can decide on which student gets which room, and how much rent will each student pay, under the

constraint that each student gets exactly one room, and the sum of payments is exactly T.

Assumptions:

1. For each student i and room r, $0 \leq v_i(r) \leq T$.
2. For each student i, $\sum_r v_i(r) \geq T$.
3. Students have quasi-linear utilities: if student i gets room r and pays p_i, then her utility is $v_i(r) - p_i$.

Propose mechanisms for the above setting. State and prove those nice properties that they have that may convince the students to use the mechanism, and also state shortcomings that they might have.

Here are some points that I expect to see addressed in the answer. They are not meant to encourage you to propose a large number of mechanisms, but rather to serve as considerations to keep in mind when designing mechanisms. You are welcome to introduce additional considerations that you think are important. *An answer with a small number of attractive mechanisms that jointly cover all points is preferable over an answer with many ad-hoc mechanisms that each covers only one point.*

1. Mechanisms may either be deterministic or randomized. If you propose a deterministic mechanism, you are expected to consider its ex-post properties. If you propose a randomized mechanism, you are expected to consider both its ex-ante properties and its ex-post properties.
2. In each of the mechanisms that you propose, might there be positive transfers (an agent receiving payment instead of making a payment)? At least one of your proposed mechanisms should not have positive transfers (ex-post). In other mechanisms, positive transfers are allowed, as long as the sum of payments (taking into account also the sign of payments, positive or negative) is exactly T.
3. Is the mechanism individually rational (IR), meaning that each student gets non-negative utility? At least one of your proposed mechanisms should be IR (ex-post).
4. Is the mechanism incentive compatible (is it a dominant strategy, or perhaps an equilibrium, for players to report their true valuations)? It

is okay to assume that agents report their valuations truthfully even if the mechanism is not incentive compatible, but at least one of your proposed mechanisms should be incentive compatible.

5. Does the mechanism maximize welfare? (Note that this depends only on the allocation but not on the payments, as payments always sum up to T.) At least one of your proposed mechanisms should maximize welfare.

6. Is the solution envy free (no agent wants to swap her room and payment with another agent)? At least one of your mechanisms has to be envy free. This requires that you prove that solutions that are envy free (ex post!) always exist.

7. Can the allocation and payments be computed in polynomial time? If so, show an algorithm. If not, can you give convincing arguments why this is a computationally difficult problem?

8. Suppose that there are two (or more) candidate apartments, each with n rooms, and the students need to jointly select one of the apartments as the one that they rent, and then apply the mechanism on the selected apartment. A rule for selecting an apartment will be called *consistent* if no agent can derive greater utility if instead a different apartment is selected, and the mechanism is employed on that other apartment. At least one of your mechanisms should have a consistent selection rule.

9. Is it really necessary to offer several mechanisms? Show some impossibility results: examples of instances in which certain combinations of desirable properties are not achievable. (For example, can you show an instance in which every envy free solution requires a positive transfer?)

The assignment was formulated in a way that forces you to propose several mechanisms. If you had to choose one mechanism to use in practice, which would you choose and why? (There is no "correct" answer here, hence the requirement to explain why you chose a particular mechanism.)