Algorithmic Game Theory - handout4

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Course webpage: https://www.wisdom.weizmann.ac.il/∼feige/agt2024.html

Homework assignments are an integral part of the course and will be a significant part of the grade. Please hand in the written assignments two weeks after they are given.

Teaching assistant: **Yotam Gafni**.

Homework. (Please keep the answers short and easy to read.)

Selfish rounting.

1. Prove that if C is the set of cost functions of the form $c(x) = ax + b$ with $a, b \geq 0$, then the Pigou bound $\alpha(C)$ is $\frac{4}{3}$. (Exercise 11.1 in [Roughgarden16]. Hint available in book.)

2. Prove that if C is the set of cost functions that are nonnegative, nondecreasing and concave (non-positive second derivative), then the Pigou bound $\alpha(C)$ is $\frac{4}{3}$. (Exercise 11.2 in [Roughgarden16]. Hint available in book.)

3. Consider a *multicommodity* directed network $G(V, E)$ where for each $i = 1, 2, \ldots k$, r_i units of traffic travel from a source $s_i \in V$ to a destination $d_i \in V$. (It is assumed that for every i, there is at least one directed path from s_i to d_i in G.) Extend the theory of the price of anarchy of equilibrium flows that we developed for the single source single destination case to the multicommodity flow case, showing that for every class C of cost functions, the Pigou bound $\alpha(C)$ remains an upper bound on the price of anarchy. (Exercise 11.5 in [Roughgarden16].)

References

[Roughgarden16] Tim Roughgarden. Twenty Lectures on Algorithmic Game Theory. Cambridge University Press, 2016.