Algorithmic Game Theory - handout5

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Course webpage: https://www.wisdom.weizmann.ac.il/~feige/agt2024.html

Homework assignments are an integral part of the course and will be a significant part of the grade. Please hand in the written assignments two weeks after they are given.

Teaching assistant: Yotam Gafni.

Homework. (Please keep the answers short and easy to read.)

Fair division. Consider a setting with $m \ge 1$ indivisible items and $n \ge 2$ agents with additive valuations, in which allocations are required to allocate all items.

- 1. Prove that if n = 2 and valuations are such that there is an envy free allocation, then every allocation that Pareto dominates an envy free allocation is also envy free.
- 2. Show that if n = 3, there are allocation instances that have an envy free allocation, but for which there is no Pareto optimal allocation that is envy free.
- 3. Prove that every EF1 allocation gives each agent at least a $\frac{1}{n}$ fraction of her MMS. Show an example in which in an EF1 allocation, some agent (with positive MMS) gets only a $\frac{1}{n}$ fraction of her MMS and no better. (Prove both statements for every n.)
- 4. Prove that for every n, every EFX allocation gives each agent at least a $\frac{n}{2n-1}$ fraction of her MMS. Show an example with two agents in which in some EFX allocation, one of the agents (who has positive MMS) gets only a $\frac{2}{3}$ fraction of her MMS and no better.