Algorithmic Game Theory - handout6

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Course webpage: https://www.wisdom.weizmann.ac.il/~feige/agt2024.html

Homework assignments are an integral part of the course and will be a significant part of the grade. Please hand in the written assignments two weeks after they are given.

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Homework. (Please keep the answers short and easy to read.)

1. In instances of fair allocation of indivisible items, consider an agent with entitlement $\frac{1}{2}$.
   (a) Prove that if the agent has an additive valuation function, then the APS equals the MMS.
   (b) Give an example of a non-additive valuation function for which the APS is strictly larger than the MMS.
2. Consider an allocation instance with n agents and a set $\mathcal{M}$ of items. Each agent i has entitlement b_i, and valuation function v_i that is described as follows. v_i has an associated additive valuation u_i that satisfies $u_i(\mathcal{M}) = 1$ and $u_i(e) \leq b_i$ for every item $e \in \mathcal{M}$. Then v_i is defined as $v_i(S) = \min[b_i, u_i(S)]$ for every set $S \subseteq \mathcal{M}$. (This is a special case of a *budget additive* valuation.) Show that there is a distribution over allocations in which each agent i gets a value of at least $\frac{3}{4}b_i$ in expectation. (Hint: see the faithful implementation lemma, slide 61 in the slides for the lecture from June 25.)
3. The VCG mechanism is incentive compatible, meaning that submitting the true valuation function is a dominant strategy for quasi-linear bidders. However, this does not mean that VCG auctions cannot be manipulated by the bidders.
   (a) Collusion. Show an example in which two bidders can coordinate their bids so that each of them would still get exactly the same set of items that she would get if both bid truthfully, but they each pay less than what they would pay under truthful bidding.

(b) False name bids. Suppose that a bidder can create several "clones" of herself, each participating in the auction under a different name. At the end of the auction, she collects all items received by her clones, and pays all their payments. Show an example in which such cloning helps the bidder get strictly higher unitily (value minus payments) compared to what she can get without cloning. (Hint: valuation functions need not be additive.)

4. Consider a connected graph $G(V, E)$ in which each edge $e \in E$ is owned by a different agent a_e (there are $|E|$ agents). Each agent a_e has a value $v_e \geq 0$ for her edge e, which is private information known only to her. If she sells the edge at price p_e then her profit is $p_e - v_e$, and she wishes to maximize her profit in the auction that follows. It is known that there is an upper bound q such that $v_e \leq q$ for all edges.

   There are two distinguished vertices $s, t \in V$, and a buyer wishes to buy a set of edges that would serve as a path P from s to t. The buyer, who does not know the v_e values, wants a path P of minimum $\sum_{e \in P} v_e$ value (subject to connecting between s and t). The buyer does not care how much she pays each of the agents (though the agents do care how much they are paid).

   In an auction mechanism agents place bids and based on these bids the buyer decides which edges to buy and how much to pay each agent. Design an auction mechanism in which the outcome (if agents do not collude and are rational) is that indeed the buyer gets a path of minimum value.