

# Twelfth Night Workshop in Representation Theory

6/1/2014– 8/1/2014

Lectures will take place in Room 261 or in Room 1, Ziskind building.

## PROGRAMME

**Monday, January 6, Afternoon**

**14:00-15:00 Ivan Penkov** (Bremen) Room 1

”A generalization of Harish-Chandra’s discrete series”

Abstract: In the seventies G. Zuckerman gave a purely algebraic description of Harish-Chandra’s discrete series. During the last decade Zuckerman, Serganova and I have been developing a theory of generalized Harish-Chandra modules. This theory studies  $(\mathfrak{g}, \mathfrak{k})$ -modules for not necessarily symmetric subalgebras  $\mathfrak{k} \subset \mathfrak{g}$ . In this talk I will give a partial classification of simple  $(\mathfrak{g}, \mathfrak{k})$ -modules of finite type for a class of subalgebras  $\mathfrak{k}$  which we call *eligible*. Symmetric root subalgebras are eligible. Our theorem concerning the fundamental series of  $(\mathfrak{g}, \mathfrak{k})$ -modules for an eligible subalgebra  $\mathfrak{k}$  can be viewed as a generalization of results of Harish-Chandra for the discrete series.

**15:00- 15:30** Tea

**15:30-16:30 Anna Melnikov** (Haifa) Room 1

” On the structure of smooth components of a Springer fiber”

Abstract: We show that a smooth component of a Springer fiber over an element of square 0 is an iterated bundle of Grassmannian varieties. We use this to compute Poincare polynomial of such component and to show that Poincare polynomials of two smooth components coincide if and only if they can be presented as a same iterated bundle. A joint work with L. Fresse.

**16:40-17:40 Polyxeni Lamprou** (Haifa) Room 1

”Quantum generalized Harish-Chandra isomorphisms”

Abstract: Let  $\mathfrak{g}$  be a semisimple Lie algebra,  $\mathfrak{h}$  a Cartan subalgebra of  $\mathfrak{g}$ . As is well-known, the Harish-Chandra projection  $p : U(\mathfrak{g}) \rightarrow U(\mathfrak{h})$  maps the  $\mathfrak{g}$ -invariants in  $U(\mathfrak{g})$  isomorphically to the  $W$ -invariants (under the translated action of  $W$ ) in  $U(\mathfrak{h})$ . Recently, Khoroshkin-Nazarov and Vinberg extended this result and described the image of the  $\mathfrak{g}$ -invariants (under the adjoint action of  $\mathfrak{g}$ ) in  $V \otimes U(\mathfrak{g})$ , where  $V$  is a finite dimensional  $\mathfrak{g}$ -module; it involves the Zhelobenko operators. Joseph gave a different proof of their result; the key point of his proof was the computation of certain determinants similar to the Parthasarathy-Ranga Rao-Varadarajan determinants. Later, Balagovic obtained an analogue of this in the quantum case. We will give alternative proofs of the Harish-Chandra isomorphisms in the semisimple and quantum case; these proofs are direct and involve simple  $\mathfrak{sl}_2$ -calculations.

**Tuesday, January 7th, Morning**

**10:00-11:00 Vyacheslav Futorny (São Paulo) Room 1**

”Classification of irreducible weight modules over Lie algebra of vector fields on a torus”

Abstract: We will discuss a classification of all irreducible weight modules over Lie algebra of vector fields on any dimensional torus, or equivalently of the Witt algebra. This is a joint result with Y. Billig (Canada). It generalizes a classical result of O. Mathieu for Virasoro algebra.

**11:00-11:20** Coffee

**11:20-12:20 Crystal Hoyt (Haifa) Room 1**

”Kac-Wakimoto character formula for  $\mathfrak{gl}(m|n)$ ”

Abstract: In 1994, Kac and Wakimoto conjectured a character formula for certain finite dimensional simple modules of basic Lie superalgebras, which specializes to the well-known Kac-Weyl character formula when a module is typical and to the Weyl denominator identity when the module is trivial. We recently proved the Kac-Wakimoto character formula for the general linear Lie superalgebra  $\mathfrak{gl}(m|n)$ , by using specified sequences of odd reflections on arc diagrams to ”move” a more general formula to the standard base of  $\mathfrak{gl}(m|n)$ , where Kazhdan-Lusztig polynomials provide an answer. We also obtained an explicit description of the highest weights of these modules in the standard base, and found that covariant modules are included. Covariant modules are by definition the irreducible components of the tensor module of the natural representation, and their characters are the super-Schur functions. In this talk, we will discuss these results and some consequences, beginning with a recollection of the root system of  $\mathfrak{gl}(m|n)$ . Joint with M. Chmutov and S. Reif.

**12:30-13:30 Inna Entova-Aizenbud (Boston) Room 261**

”Schur-Weyl duality in complex rank”

Abstract: Let  $V$  be a finite dimensional vector space. The classical Schur-Weyl duality describes the relation between the action of the Lie algebra  $\mathfrak{gl}(V)$  and the symmetric group  $S_n$  on the tensor power  $V^{\otimes n}$ . We will discuss Deligne categories  $Rep(S_t)$ , which are extrapolations to complex  $t$  of the categories of finite dimensional representations of the symmetric groups. I will then present a generalization of the classical Schur-Weyl duality in the setting of Deligne categories, which involves a construction of ”a complex tensor power of  $V$ ”, and gives us a duality between the Deligne category and a Serre quotient of a parabolic category  $\mathcal{O}$  for  $\mathfrak{gl}(V)$ .

**13:50-15:30** Lunch in Cafe Mada

**Tuesday, January 7th, Afternoon**

**15:40-16:40 Vera Serganova** (Berkeley) Room 1

”Representations of the Lie superalgebra  $P(n)$  and Brauer algebras with signs”

Abstract: The ”strange” Lie superalgebra  $P(n)$  is the algebra of endomorphisms of an  $(n|n)$ -dimensional vector space  $V$  equipped with a non-degenerate odd symmetric form. The centralizer of the  $P(n)$ -action in the  $k$ -th tensor power of  $V$  is given by a certain analogue of the Brauer algebra. We discuss some properties of this algebra in application to representation theory of  $P(n)$  and  $P(\infty)$ . We also construct a universal tensor category such that for all  $n$  the categories of  $P(n)$  modules can be obtained as quotients of this category. In some sense this category is an analogue of the Deligne categories  $GL(t)$  and  $SO(t)$ .

**16:50-17:50 Iryna Kashuba** (Saõ Paulo) Room 261

”Deformations of Jordan algebras”

Abstract: Let  $k$  be an algebraically closed field,  $n$  be a positive integer and  $\mathbb{A} = \mathbb{A}_k^{n^3}$  be an  $n^3$ -dimensional affine space. Any point  $c = \{c_{ij}^h\}$  of  $\mathbb{A}$  gives a collection of structure constants defining a  $k$ -algebra. The set of all points of  $\mathbb{A}$  such that the product is Jordan defines an algebraic subvariety  $Jor_n \subset \mathbb{A}$ . The group  $GL_n$  acts on  $Jor_n$  by so-called ”transport of structure” action, and  $GL_n$ -orbits of this action are in one-to-one correspondence with the isomorphism classes of  $n$ -dimensional Jordan algebras. An algebra  $J_2$  is called a deformation of an algebra  $J_1$  if the orbit  $J_1^{GL_n}$  belongs to the Zariski-closure of the orbit  $J_2^{GL_n}$ . We will write down basic properties and useful characteristics of  $Jor_n$ , give complete description for small  $n$  and will talk about estimate of the dimension of  $Jor_n$ .

**18:00** Buffet Supper (Room 141)

## Wednesday, January 8th, Morning

**9:30-10:30 Joseph Bernstein** (Tel Aviv) Room 1

”Stacks in Representation Theory”

**10:30-10:50** Coffee

**10:50-11:50 Avraham Aizenbud** (Rehovot) Room 261

” $\mathfrak{Z}$ -finite distributions on p-adic groups”

Abstract: In the Archimedean case, the study of  $\mathfrak{Z}(U(\mathfrak{g}))$ -finite distributions on a real reductive group  $G$  had many applications in representation theory and particularly in the study of characters and spherical characters. The natural analog of the center  $\mathfrak{Z}(U(\mathfrak{g}))$  of the universal enveloping algebra for the non-Archimedean case is the Bernstein center  $\mathfrak{Z}(G)$ . However, since there is no good geometric description of the Bernstein center, there were no results on  $\mathfrak{Z}$ -finite distributions on p-adic groups, till now.

I will present two recent results on such distributions:

- 1) A bound on the wave front set of such distributions (similar to the standard bound on the characteristic variety of  $\mathfrak{Z}(U(\mathfrak{g}))$ -finite distributions in the Archimedean case).
- 2) Density of  $\mathfrak{Z}(G)$ -finite distributions inside some spaces of invariant distributions.

While the first result is standard in the Archimedean case, the second is still an open problem in this case.

**12:10-13:10 Anthony Joseph** (Rehovot) Room 261

”Relative Yangians of Weyl type”

Abstract: Let  $\mathfrak{g}$  be a complex simple Lie algebra,  $V$  a finite dimensional  $\mathfrak{g}$  module and  $D(V)$  the algebra of differential operators on  $V^*$ . The relative Yangian  $E_V(\mathfrak{g})$  of Weyl type with respect to the pair  $\mathfrak{g}, V$  is defined to be the  $\mathfrak{g}$  invariant subalgebra of  $D(V) \otimes U(\mathfrak{g})$  with respect to the natural ”diagonal” action of  $\mathfrak{g}$ .

An abelian category  $\mathcal{E}^*$  of  $E_V(\mathfrak{g})$  modules is described and in which the simples and their projective covers determined. The annihilators of the simples do not quite recover all the primitive ideals of  $E_V(\mathfrak{g})$ .

When  $D(V)$  is replaced by  $EndV$  such algebras were called ”family algebras” by Kirillov, who studied their structure but not their representation theory, now simply called relative Yangians. They include relative Yangians of Clifford type (for which  $D(V)$  is replaced by the Clifford algebra  $C(V)$  of  $V$ ).

This is a continuation of our earlier work on the representation theory of relative Yangians in which all simple modules were described. It completed work of Nazarov and

Khoroshkin, notably extending their results for  $\mathfrak{g}$  classical and in this taking a rather different point of view.

The interest of these algebras lies partly in the Olshanski homomorphism to Yangians and partly in the fact that the Kazhdan-Lusztig polynomials are recovered as homological data.

**13:40-15:50** Lunch in Cafe Mada

**Wednesday, January 8th, Afternoon**

**16:00-17:00 Daniel Fleisher** (Rehovot) Room 1

**17:10-18:10 Andrey Minchenko** (Rehovot) Room 1

”Semisimple subalgebras of real Lie algebras”

Abstract: For a complex or a real Lie algebra  $\mathfrak{g}$ , let  $S[\mathfrak{g}]$  denote the set of its semisimple subalgebras. Let  $G$  be a Lie group and  $\mathfrak{g}$  be its Lie algebra. The complexification  $G \mapsto G_{\mathbb{C}}$  induces the map  $S[\mathfrak{g}]/G \rightarrow S[\mathfrak{g}_{\mathbb{C}}]/G_{\mathbb{C}}$  of conjugacy classes. I will explain how to find the fibers of this map.

**18:10** Tea (Room 141)