HOMEWORK 2

The base field is $\mathbb{C}$. For a superalgebras A, B we view $A \otimes B$ as a superalgebra via

$$(a \otimes b) \cdot (a' \otimes b') := (-1)^{d(b)d(a')} aa' \otimes bb'.$$

1. Let $\mathfrak{g}_1, \mathfrak{g}_2$ be Lie superalgebras. Check that $\mathfrak{g}_1 \otimes \mathfrak{g}_2$ is not a Lie superalgebra.
2. Describe the tensor product of two Clifford algebras $C(m), C(n)$. For a superalgebra A denote by A' the corresponding non-graded algebra. Show that $\big(C(m) \otimes C(n)\big)'$ and $C(m)' \otimes C(n)'$ are non-isomorphic.
3. Let $\mathfrak{g}_1, \mathfrak{g}_2$ be Lie superalgebras. Check that
$$\mathcal{U}(\mathfrak{g}_1 \times \mathfrak{g}_2) = \mathcal{U}(\mathfrak{g}_1) \otimes \mathcal{U}(\mathfrak{g}_2).$$
4. Let $\mathfrak{g} := H(n)$ be the Heisenberg superalgebra and $V(c)$ be its simple module. Find $\mathrm{End}_{\mathfrak{g}} V(c)$.
5. The algebra $\mathfrak{psl}(n|n)$ is the quotient of $\mathfrak{sl}(n|n)$ by the one-dimensional centre (spanned by the identity matrix). Show that the Killing form $K(g_1, g_2) := \mathrm{str}(\mathrm{ad} g_1 \circ \mathrm{ad} g_2)$ for $\mathfrak{psl}(n|n)$ iz zero. Find a non-degenerate invariant bilinear form on $\mathfrak{psl}(n|n)$.
6. Find all Cartan matrices for $\mathfrak{sl}(2|1)$.