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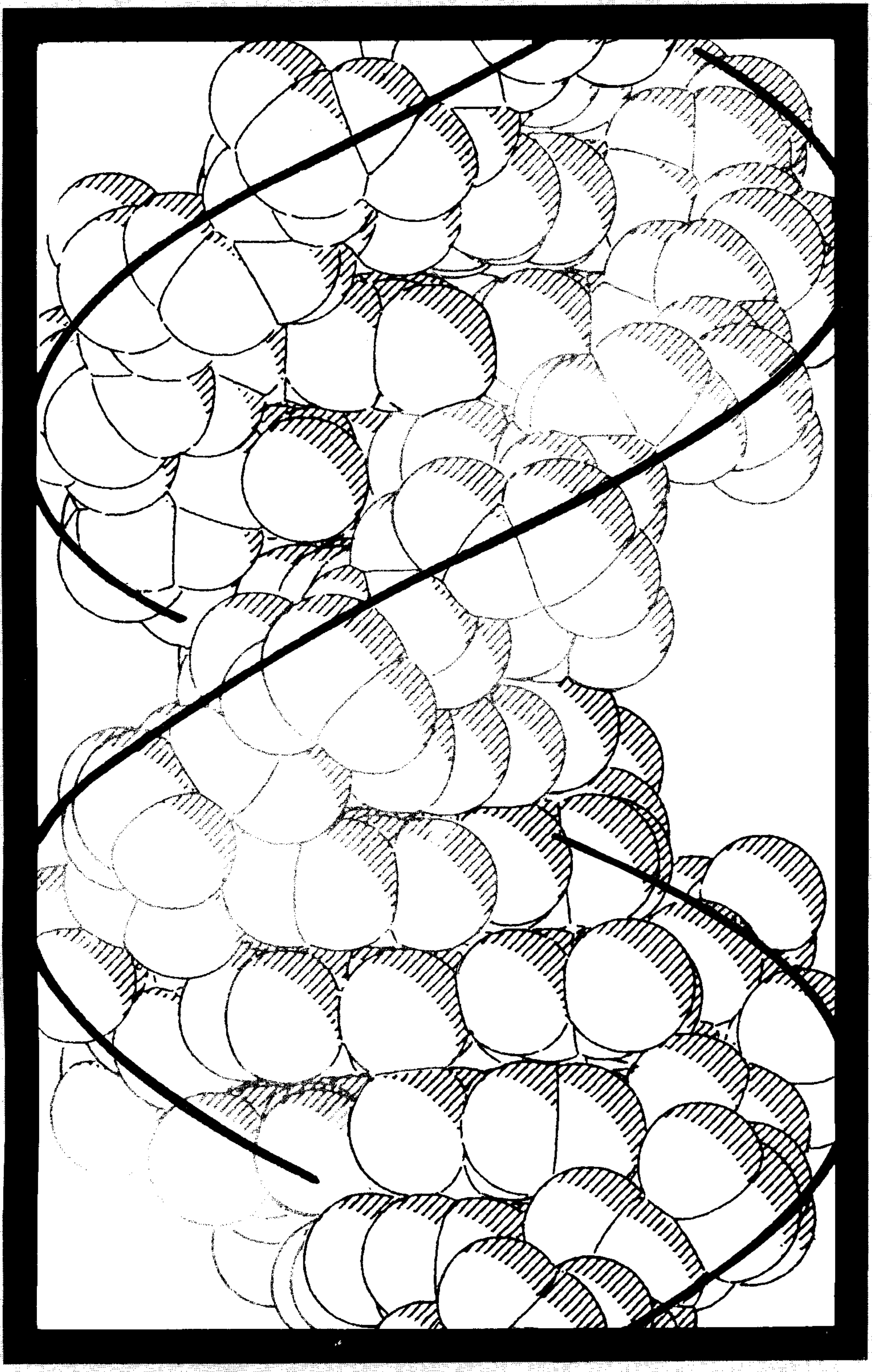

Features

Beauty is in the genes of the beholder*

D. Harel, R. Unger and J. L. Sussman

An interpretation of the particular proportions of DNA as found in the Watson–Crick double helix model is suggested. It is based on the classical, highly aesthetical concept known as the Golden Ratio. *Specifically, it is shown that in B-DNA both pitch/diameter and diameter/offset are extremely close to the Golden Ratio. Here, pitch is the helical repeat, and offset refers to the vertical distance that forms the minor groove.*

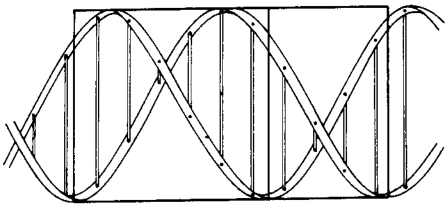

Fig. 1. Golden Ratio in DNA.

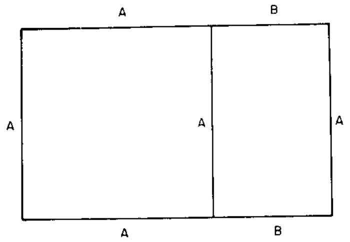

Fig. 2. Schematic illustration of Golden Ratio: (A+B)/A = A/B, *so that both the external and internal rectangles are identically proportioned.*

We wish to suggest an interpretation of the particular proportions of DNA as found in the Watson–Crick double helical model[1]. This interpretation has novel features which are of considerable historical, theological, mathematical and, above all, aesthetical interest.

The Watson–Crick model, generally referred to as *the double helix*, consists of two helical chains coiled round the same axis. These chains (excluding the bases) are related by a dyad perpendicular to the helix axis; thus the two chains are antiparallel (see Fig. 1).

There have been many attempts of biological and chemical nature to explain why this particular structure is the one DNA abides by. In our opinion, these explanations are not wholly satisfactory for various reasons. We believe that a definitive, universally acceptable, historically rooted and highly intuitive interpretation should exist for the particular structure and proportions of this most profound and fundamental molecule.

We wish to put forward a radically different interpretation of the structure for the salt of deoxyribose nucleic acid. This interpretation is concerned with the internal dimensionality and proportions of the double helix; it can be thought of as involving two of the most appealing branches of human intellect, mathematics and art, both coiled round the same axis, that of divine aesthetics.

We submit that DNA is structured as it is mainly for aesthetic reasons and support this claim with the use of the Golden Ratio, a concept going back to ancient times. The Golden Ratio, denoted ϕ, is an irrational number, like π, which expresses a fundamental ratio that is almost as common as π and has the habit of appearing when least expected[2]. The geometrical meaning of ϕ is seen from the line segments of Fig. 2. The horizontal length of the rectangle has been divided into two parts, A and B, such that the ratio of B to A is the same as that of A to $A+B$, the length of the entire line. In each case the ratio can easily be shown to be given by:

$$\phi = \frac{1+\sqrt{5}}{2}$$

or approximately 1.61803398.

There is little doubt that the Ancient Greeks were familiar with the Golden Ratio and it was used by some of their architects and sculptors, particularly in the structure of the Parthenon (see Fig. 3a). In fact the name ϕ was originally given by the American mathematician Mark Barr in the early 1900s in memory of the famous Greek sculptor Phidias who often used the Golden Ratio in his work[2].

The number ϕ appears in Renaissance art, including works of da Vinci and others, and is rumoured to be the basis of the segmentation of a well-built human body; first finger joint to second, second to both, hand to lower arm, lower arm to hand+lower arm, and so on, culminating in navel upwards to navel downwards and navel downwards to whole body. This particular observation strengthens

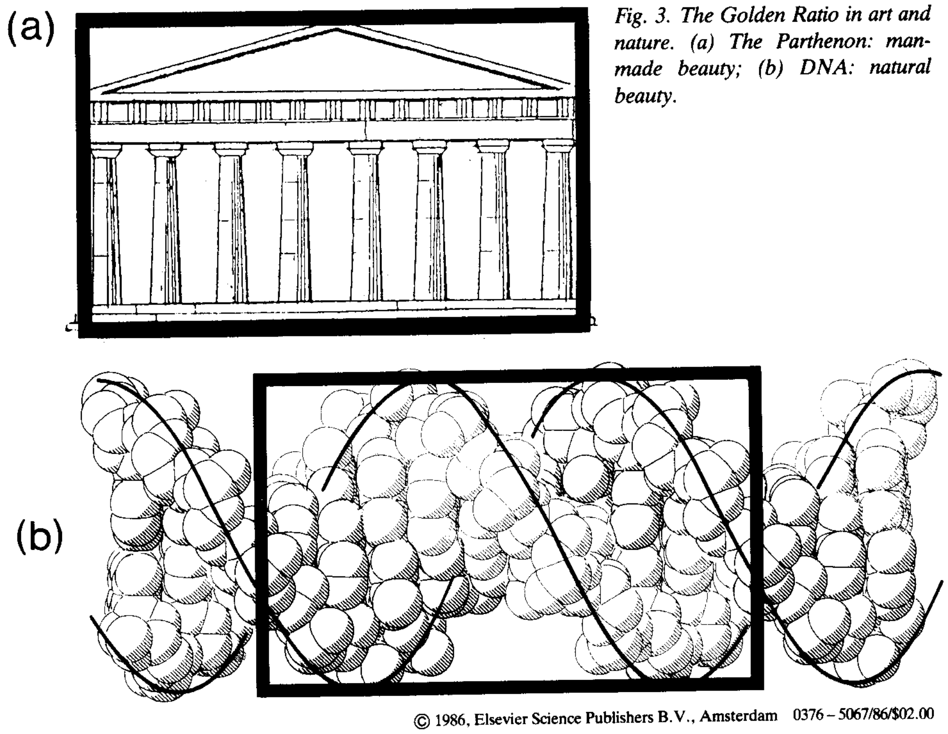

Fig. 3. The Golden Ratio in art and nature. (a) The Parthenon: man-made beauty; (b) DNA: natural beauty.

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*With apologies to Watson and Crick.

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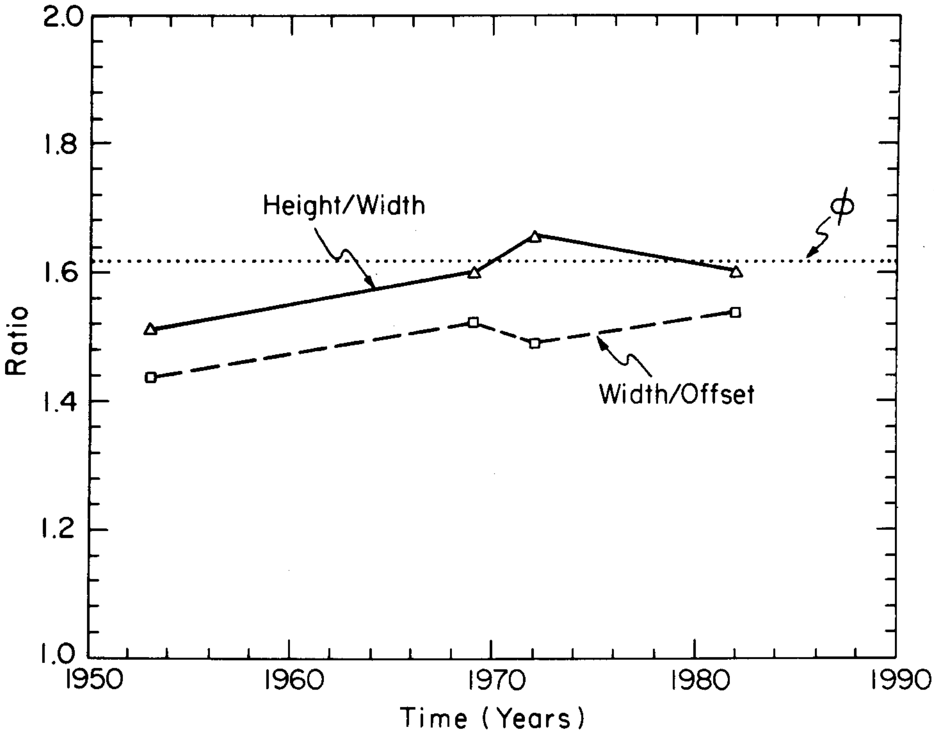

Fig. 4. Beauty improves in time. The coordinates for the 4 DNA structures are taken from Refs 5 (re-refined by Arnott, S and Chandrasekaran, R., personal communication, 1982) and 6–8.

one's feeling that the proportions of DNA should similarly be based on φ.

That the structure of a double helix is in itself one of immense beauty goes without saying; we concentrate here on the local dimensions of the type-B double helix, which appears to be the most common form of DNA in chromatin[3]. It is easy to conceive of a double helix of uncomely proportions, but it is difficult to imagine that one which was unpleasant to behold would be chosen to carry the genetic code. Given that DNA is to be structured as a right-handed double helix, there are three crucial dimensions that essentially determine its final form: the external *width* (diameter) of the double helix, the length of its period (i.e. the *pitch*, or the *height* of a representative 'slice'), and the vertical *offset* of one helix from the other, which forms the minor groove.

The novel feature of our interpretation of Watson and Crick's structure is the manner in which these three basic dimensions are held together by the rule of the Golden Ratio. Specifically we have found that the ratio of the height to the width, and the ratio of the width to the offset are always very close to φ (see Figs 1 and 3). In particular, since the ratio between height and offset is responsible for the unequal sizes of the major and minor grooves of B-DNA, we might say that the ratio between the grooves themselves is golden too.

Although the crystal structure of a B-DNA dodecamer[4] has been refined to high resolution, it is too short and irregular to accurately estimate the pitch, diameter and offset of the DNA molecule as a whole. However the most recent and best refined X-ray fibre studies of long chains of B-DNA[5] (re-refined by Arnott, S. and Chandrasekaran, R., personal communication 1982) yield ratios of 1.6031 and 1.538, respectively, or approximately 1 and 5% less than φ. Moreover, both ratios seem to converge to φ as time proceeds (Fig. 4). In contrast, in the less frequently occurring forms of DNA, i.e. A-DNA and Z-DNA, the ratios are further from φ, as shown in Table I.

Currently available X-ray data on DNA are insufficient for a rigorous test of our interpretation of the structure. As far as we can tell, however, our interpretation is smoothly compatible with the experimental data (and is getting better with time*), but it must be regarded as unproven until it has been checked against more exact results. Our 99 and 95% fits, in other words, are only the starting point . . .

It has not escaped our notice that the specific interpretation we have postulated immediately suggests a possible motto for the copying mechanism of the genetic material:

Beauty is in the genes
of the beholder

Table I. Models of DNA Structures and their fit to the Golden Ratio.

Model	Height (Å) (pitch)	Width (Å)[a] (diameter)	Offset(Å)[b]	Height/width	Width/offset
A-DNA[5],C	25.6	18.9	19.1	1.35	0.99
B-DNA[5],C	33.8	21.1	13.7	1.60	1.54
Z-I DNA[9]	44.6	17.5	15.9	2.55	1.10
Z-II DNA[9]	44.6	18.3	10.9	2.44	1.68

[a] Width determined by the atom most distant from the helix axis: A-DNA O5′, B-DNA O2P, Z-I DNA C8 (G), Z-II DNA O2P (C).

[b] Offset determined by vertical separation between the two helices of the double helix based on the locus of phosphorus atoms.

[c] Re-refined by Arnott, S. and Chandrasekaran, R., personal communication (1982).

Acknowledgement

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***Note added in proof**

In fact, just 7 years after the Watson–Crick model was proposed Langridge *et al.*[10] built a DNA model with ratios of height/width = 1.63, and width/offset = 1.51.

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