Is there an infinite expander?

Weak limits of many examples of d-regular expanders with growing girths are infinite d-regular trees. Any tree can be divided into two infinite sets with only one boundary edge between them.

For a set of vertices S in a graph G denote it's boundary by ∂S. Recall that the boundary of a set S consists of vertices not in S, with a neighbor in S.

A uniformly bounded degree connected graph G is called an **infinite expander** if there is $c > 0$ so that for any set S of vertices finite or infinite and any ball $B(r), r < \infty$ in G, for which $|S \cap B(r)| < |B(r)|/2$,

$$|\partial S \cap B(r)| > c|S \cap B(r)|.$$

Conjecture 0.1. *There is no infinite expander.*