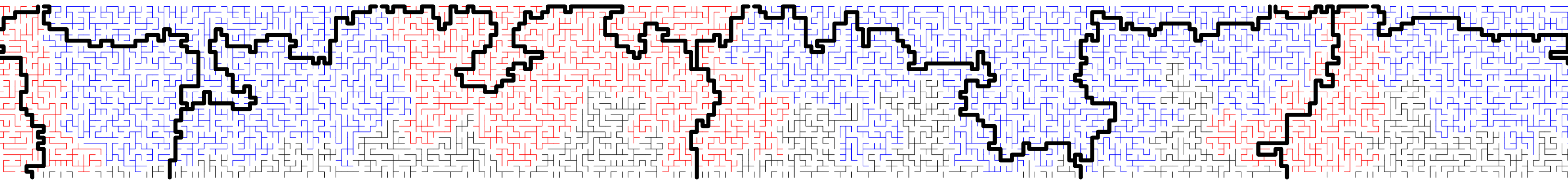
Wanted



Consider a uniform spanning tree in a square grid horizontal strip $\mathbb{Z} \times \{0, 1, 2, ..., n\}$, where the lower boundary is wired together and the upper boundary is “free”. Each component of the complement of the wired vertex is a “bush”, and a bush that has vertices on the top row is a “crossing bush”. In the above picture, the crossing bushes are colored red and blue. The root of a crossing bush is the edge that connects it to the wired vertex. The leftmost and rightmost paths from the root to the upper boundary are indicated in the figure by bold black lines. The collection of roots of crossing bushes is a stationary point process on $\mathbb{Z}$. Let $n \to \infty$, rescale by n, and in the limit get a stationary point process on $\mathbb{R}$ (it can be shown that the limit exists, that is, there's no need to take a subsequence). One can ask many questions about the limiting process, but the most basic one is to find the distribution of the length of a segment between two adjacent points. Using the techniques developed by Rick Kenyon, it can be proven that the density of that distribution is

$$\frac{2e^{x}\left(1 - e^{2x} + (1 + e^{2x})x\right)}{\left(e^{2x} - 1\right)^{2}}\,.$$

The two most prominent features of this distribution is the linear repulsion (the density behaves like $Cx + o(x)$ near zero) and the exponential decay when x is large.

Do you recognize this distribution? Have you seen it before?

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