Algorithmic Game Theory – Handout 7

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We study inefficiency of equilibria, namely comparison between an optimal outcome of the game and an equilibrium outcome (with respect to an objective function), and its quantitative version called price of anarchy. Today's focus is price of anarchy in routing games, and in particular in nonatomic selfish routing, a multicommodity flow model where each agent is selfish and controls a negligible portion of the flow.

Reading. More information on inefficiency of equilibria can be found in [NRTV, Part III]. The material covered in class appears in Chapters 17 and 18.

Announcements.

- The exam will take place on February 25, at 2pm.
- There will be no class on Dec. 31. You are encouraged to attend the computational game theory workshop that day `http://pluto.huji.ac.il/~mfeldman/cgt4_2008.html`

Homework. Please keep the answers to the following questions short and easy to read.

1. Consider nonatomic selfish routing where edge costs can be any polynomial of degree up to p with nonnegative coefficients. First, prove that for every such network, the price of anarchy is at most $p+1$. next, show that the price of anarchy can be as large as $\Omega(p/\ln p)$, even in a simple network (Pigou's network).
2. Prove that the price of anarchy in nonatomic selfish routing under a set of nondecreasing and differentiable cost functions $\mathcal{C}$ is at most

$$\alpha(\mathcal{C}) = \sup_{c\in\mathcal{C}} \sup_{x,r\geq 0} \frac{r\cdot c(r)}{x\cdot c(x)+(r-x)c(r)}.$$

   Hint: Given a Wardrop equilibrium flow f in (G,r,c), create G' by adding for every edge e a parallel edge e' that has a constant cost $c_{e'}(x) = c_e(f_e)$. Now use the flow f to construct a flow f^* which is a Wardrop equilibrium on G' under a modified cost function $c^*_e(x) = c_e(x) + x\cdot c'_e(x)$. Use the latter to bound the cost of optimal flow in G (under the original cost c).

   Useful fact: $\frac{a+b}{a'+b'} \leq \max\{\frac{a}{a'}, \frac{b}{b'}\}$.

References

[NRTV] Noam Nisan, Tim Roughgarden, Eva Tardos and Vijay V. Vazirani (Editors), Algorithmic Game Theory, Cambridge University Press, 2007.