Information Theory: Exercise IV

1) Let G be the complete (undirected) graph with $m = 4n$ vertices. That is, $G = (V, E)$, where V is a set of size $4n$, and E contains all pairs (i, j), s.t., $i, j \in V$ and $i \neq j$.

Let H be the complete (undirected) tripartite graph with parts of sizes $2n, n, n$. That is, $H = (V_1 \cup V_2 \cup V_3, E')$, where V_1, V_2, V_3 are disjoint sets of sizes $2n, n, n$ (respectively), and E' contains all pairs (i, j), s.t., $i \in V_a, j \in V_b$, where $a \neq b$.

We say that a sequence of graphs $H_1, \ldots, H_k$ covers a graph G if the nodes of $H_1, \ldots, H_k$ can be placed on the nodes of G (i.e., the nodes of every H_i are mapped one-to-one to the nodes of G), such that, every edge of G is covered by at least one edge of $H_1, \ldots, H_k$.

Show that at least $(2/3) \cdot \log_2 m$ copies of H are needed to cover G.

2) Let G be the graph with set of nodes $\{1, 2, 3\}^n$, where two nodes $x, y \in \{1, 2, 3\}^n$ are connected by an edge iff they differ in exactly one coordinate. State and prove an isoperimetric inequality for the graph G. (Similar to the isoperimetric inequality for the discrete cube $\{0, 1\}^n$).

3) Prove or give a counter example: For every X_1, X_2, X_3, X_4,

$$H(X_1, X_2, X_3) + H(X_1, X_2, X_4) + H(X_1, X_3, X_4) + H(X_2, X_3, X_4) \leq$$

$$H(X_1, X_2) + H(X_1, X_3) + H(X_1, X_4) + H(X_2, X_3) + H(X_2, X_4) + H(X_3, X_4).$$

4) Prove or give a counter example: For every X_1, X_2, X_3, X_4,

$$H(X_1, X_2, X_3) + H(X_2, X_3, X_4) + H(X_3, X_4, X_1) + H(X_4, X_1, X_2) \leq$$

$$1.5 \cdot [H(X_1, X_2) + H(X_2, X_3) + H(X_3, X_4) + H(X_4, X_1)].$$

5) Prove or give a counter example: For every X_1, X_2, X_3, X_4,

$$H(X_1, X_2, X_3) + H(X_1, X_2, X_4) + H(X_1, X_3, X_4) + H(X_2, X_3, X_4) \leq$$

$$3 \cdot [H(X_1, X_2) + H(X_3, X_4)].$$