Distributed Summaries

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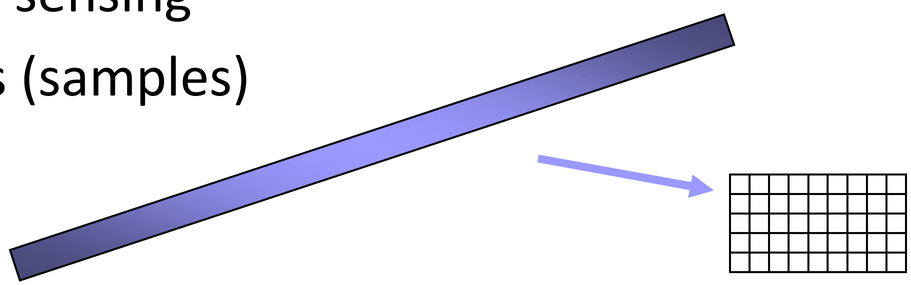
Summaries

- Summaries allow approximate computations:
  - Euclidean distance (Johnson-Lindenstrauss lemma)
  - Vector Inner-product, Matrix product (sketches)
  - Distinct items (Flajolet-Martin onwards)
  - Frequent Items (Misra-Gries onwards)
  - Compressed sensing
  - Subset-sums (samples)

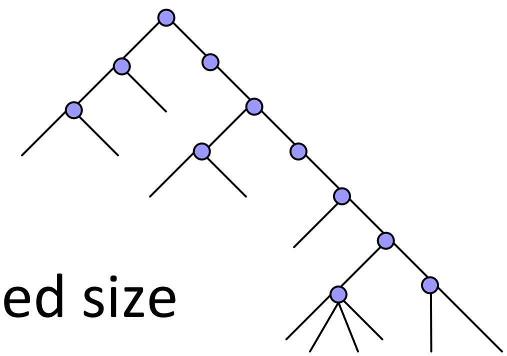


Approximation and Parallel Computation

- Why use approximate when data storage is cheap?
  - Parallelize computation: partition and summarize data
    - Consider holistic aggregates, e.g. count-distinct
  - Faster computation (only send summaries, not full data)
    - Less marshalling, load balancing needed
  - Implicit in some tools (Sawzall)

Mergability

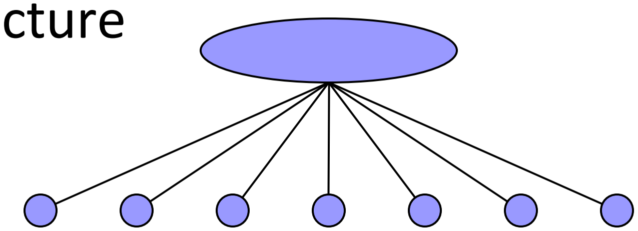
- Ideally, summaries are algebraic: associative, commutative
  - Allows arbitrary computation trees
    (see also synopsis diffusion [Nath+04], MUD model)
  - Distribution "just works", whatever the architecture



- Summaries should have bounded size
  - Ideally, independent of base data size
  - Or sublinear in base data (logarithmic, square root)
  - Should **not** depend on number of merges
  - Rule out "trivial" solution of keeping union of input

Models of Summary Construction

- Offline computation: e.g. sort data, take percentiles
- Streaming: summary merged with one new item each step
- One-way merge: each summary merges into at most one
  - Single level hierarchy merge structure
  - Caterpillar graph of merges

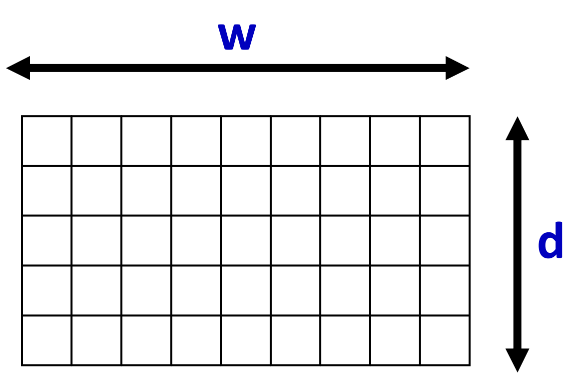


- Equal-size merges: can only merge summaries of same arity
- Full mergeability: allow arbitrary merging schemes
  - Our main interest

Merging: sketches

- Example: most sketches (random projections) fully mergeable
- Count-Min sketch of vector $x[1..U]$:
  - Creates a small summary as an array of $w \times d$ in size
  - Use d hash functions h to map vector entries to $[1..w]$
  - Estimate $x[i] = \min_j CM[h_j(i), j]$
- Trivially mergeable: $CM(x + y) = CM(x) + CM(y)$

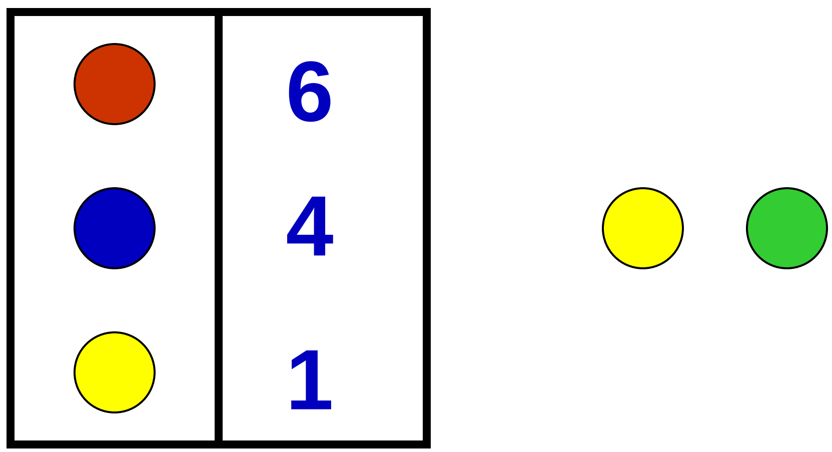
Array: CM[i,j]



Merging: sketches

- Consequence of sketch mergability:
  - Full mergability of quantiles, heavy hitters, F0, F2, dot product…
  - Easy, widely implemented, used in practice
- Limitations of sketch mergeability:
  - Probabilistic guarantees
  - May require discrete domain (ints, not reals or strings)
  - Some bounds are logarithmic in domain size

Summaries for heavy hitters



- Misra-Gries (MG) algorithm finds up to k items that occur more than 1/k fraction of the time in a stream
- Keep k different candidates in hand. For each item in stream:
  - If item is monitored, increase its counter
  - Else, if < k items monitored, add new item with count 1
  - Else, decrease all counts by 1

Streaming MG analysis

- N = total weight of input
- M = sum of counters in data structure
- Error in any estimated count at most (N-M)/(k+1)
  - Estimated count a lower bound on true count
  - Each decrement spread over (k+1) items: 1 new one and k in MG
  - Equivalent to deleting (k+1) distinct items from stream
  - At most (N-M)/(k+1) decrement operations
  - Hence, can have “deleted” (N-M)/(k+1) copies of any item

Merging two MG Summaries

- Merging alg:
  - Merge the counter sets in the obvious way
  - Take the (k+1)th largest counter = C_{k+1}, and subtract from all
  - Delete non-positive counters
  - Sum of remaining counters is M_{12}
- This alg gives full mergeability:
  - Merge subtracts at least $(k+1)C_{k+1}$ from counter sums
  - So $(k+1)C_{k+1} \leq (M_1 + M_2 - M_{12})$
  - By induction, error is
    $((N_1-M_1) + (N_2-M_2) + (M_1+M_2-M_{12}))/(k+1)=((N_1+N_2) - M_{12})/(k+1)$

Quantiles

- Quantiles / order statistics generalize the median:
  - Exact answer: $CDF^{-1}(\phi)$ for $0 < \phi < 1$
  - Approximate version: tolerate answer in $CDF^{-1}(\phi - \varepsilon) \ldots CDF^{-1}(\phi + \varepsilon)$
- Hoeffding bound: sample of size $O(1/\varepsilon^2 \log 1/\delta)$ suffices
- Easy result: one-way mergeability in $O(1/\varepsilon \log (\varepsilon n))$
  - Assume a streaming summary (e.g. Greenwald-Khanna)
  - Extract an approximate CDF F from the summary
  - Generate corresponding distribution f over n items
  - Feed f to summary, error is bounded
  - Limitation: repeatedly extracting/inserting causes error to grow

Equal-weight merging quantiles

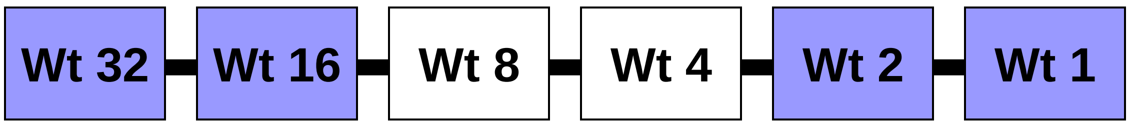
- A classic result (Munro-Paterson '78):
  - Input: two summaries of equal size k
  - Base case: fill summary with k input items
  - Merge, sort summaries to get size 2k
  - Take every other element
- Deterministic bound:
  - Error grows proportional to height of merge tree
  - Implies $O(1/\varepsilon \log^2 n)$ sized summaries (for n known upfront)
- Randomized twist:
  - Randomly pick whether to take odd or even elements

Equal-size merge analysis

- Analyze error in range count for any interval after m merges
- Absolute error introduced by i'th level merge is 2^{i-1}
- Unbiased: expected error is 0 (50-50 $+2^{i-1}$ / -2^{i-1})
- Apply Chernoff bound to sum of errors
- Summary size $= O(1/\varepsilon \log^{1/2} 1/\delta)$ gives εN error w/prob $1-\delta$
  - Neat: naïve sampling bound requires $O(1/\varepsilon^2 \log 1/\delta)$
  - Tightens randomized result of [Suri Toth Zhou 04]

Fully mergeable quantiles

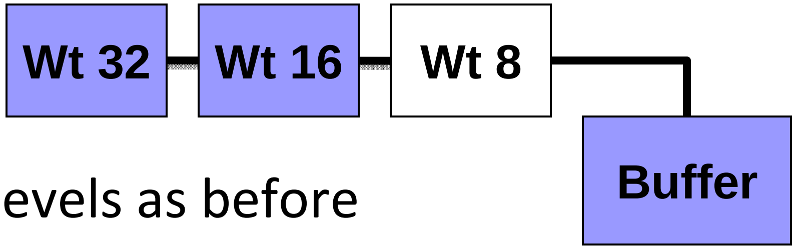
- Use equal-size merging in a standard logarithmic trick:



- Merge two summaries as binary addition
- Fully mergeable quantiles, in $O(1/\varepsilon \log (\varepsilon n) \log^{1/2} 1/\delta)$
  - n = number of items summarized, **not** known a priori
- But can we do better?

Hybrid summary

- Observation: when summary has high weight, low order blocks don't contribute much
  - Can't ignore them entirely, might merge with many small sets
- Hybrid structure:



  - Keep top $O(\log 1/\varepsilon)$ levels as before
  - Also keep a "buffer" sample of (few) items
  - Merge/keep equal-size summaries, and sample rest into buffer
- Analysis rather delicate:
  - Points go into/out of buffer, but always moving "up"
  - Gives constant probability of accuracy in $O(1/\varepsilon \log^{1.5}(1/\varepsilon))$

Other Fully Mergeable Summaries

- Samples on distinct (aggregated) keys
- ε-approximations in constant VC-dimension v in $O(\varepsilon^{-2v/(v+1)})$
- ε-kernels in d-dimensional space in $O(\varepsilon^{(1-d)/2})$
  - For "fat" pointsets: bounded ratio between extents in any direction
- Equal-weight merging for k-median implicit from streaming
  - Implies O(poly n) fully-mergeable summary via logarithmic trick

Open Problems

- Weight-based sampling over non-aggregated data
- Fully mergeable ε-kernels without assumptions
- More complex functions, e.g. cascaded aggregates
- Lower bounds for mergeable summaries
- Implementation studies (e.g. in Hadoop)