Lecture 3, February 28, 2008

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Administrative issues:

- If you wish to be a grader of the homework assignments and believe that you can do a good job, please contact us.
- Lecture notes for a course given in the past with a lot of overlap with the current course can be found at http://www.wisdom.weizmann.ac.il/ feige/algs04.html.

Today we plan to cover the following topics:

- Fourier-Motzkin elimination (a naive and inefficient way of solving linear programs, but worth keeping in mind).
- Additional remarks on the simplex algorithm. In particular, how to find a starting BFS.
- More on the notion of basic feasible solutions. A very instructive application – the Beck-Fiala theorem.

Homework

1. At a degenerate vertex, a variable with negative reduced cost might enter the basis without improving the value of the objective function. A. Hoffman designed an example showing that cycling (a sequence of changes of bases that eventually returns to the original basis) can occur in the simplex algorithm. Can the following most simple form of cycling occur at a degenerate vertex: first x_1 enters the basis and x_2 leaves it; then x_2 enters the basis and x_1 leaves it? If yes, provide an example where this happens. If no, explain why not.
2. Let G be an arbitrary graph and let $\alpha_1, \ldots, \alpha_k$ be nonnegative with $\sum_{i=1}^{k} \alpha_i = 1$. Show (we recommend using a proof similar to that of the Beck-Fiala theorem) that one can color the edges of the graph such that for every vertex v and every color class i, the number of edges of color i incident with vertex v is between $\lfloor \alpha_i d_v \rfloor - 2$ and $\lceil \alpha_i d_v \rceil + 2$, where d_v is the degree of vertex v. [Some remarks. The slackness term of 2 can be improved when $k = 2$, and it is an open question whether it can be improved in general. For the special case of bipartite graphs, if $k = 2$ no slackness term is needed at all, and it is conjectured that this is true for every k.]