Advanced Algorithms – Handout 5

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1 Today's topics

The geometry of LP revisited – polyhedral theory:

- Polyhedra and polytopes
- Faces, facets and extreme points.
- Characterizing a polytope as a convex hull of its extreme points

2 Homework

1. Let $P = \{x \in R^n : \ Ax \leq b\}$. Show that $x \in P$ is an extreme point of P iff it is a vertex of P.

   Recall: (i) x is an extreme point if and only if there is $c \in R^n$ such that x is the unique optimum for $\min\{c^t x : \ x \in P\}$. (ii) x is a vertex if there is no $y \neq 0$ such that both $x + y, x - y \in P$, i.e. it is not a convex combination of two other points in P.

   Hints: If x is an extreme point, write it as a unique solution to a subset of constraints. If x is a vertex, build intuition by thinking about the case that x^* satisfies some subset I of constraints of the form $a^t x \leq 1$ with equality.

2. Let x, y be two extreme points of $P = \{x \in R^n : \ Ax \leq b\}$. Show that x, y are *adjacent* (in the sense that there is an edge, i.e. a face of dimension 1, containing both x, y) if and only if there is a *unique* way to express their midpoint $\frac{1}{2}(x + y)$ as a convex combination of extreme points of P.