Algorithm course, last class

May 29, 2008

Topics covered in course (not necessarily in this order).

- Algebraic formulation of linear programs, standard form, canonical form. Fourier-Motzkin elimination.
- Geometric view of linear programs. Basics of polyhedral theory, polyhedrons, polytopes, hyperplanes, convex sets, extreme points, facets.
- Basic feasible solutions, bounds on precision.
- Beck-Fiala Theorem.
- The simplex algorithm, pivoting rules, reduced costs, finding a feasible starting point.
- LP Duality, formulating the dual, weak duality, strong duality, complementary slackness, Farkas lemma, Helly's theorem.
- Ellipsoid algorithm, geometric view, positive definite matrices, bounding ball, bounded ball, separation oracles.
- Generalization beyond linear programming, semidefinite programming, low distortion embeddings into Euclidean space.
- LP relaxations, integrality gaps, total unimodularity, half-integrality, randomized rounding (set cover, max-sat).
- Perfect graphs, classes of perfect graphs, the Lovasz theta function, finding a maximum independent set in perfect graphs, computing the chromatic number and finding a corresponding coloring.
- Multiplicative Weights method and solving LPs approximately (survey by Arora, Hazan, Kale).
- Approximating max-cut using SDP (Goemans and Williamson).
- Max-cut in planar graphs, the chinese postman problem.

- Coloring 3-colorable graphs with small number of colors (Karger-Motwani-Sudan).

A taste of well known open questions.

1. The simplex algorithm.

   - Hirsch's conjecture: does the skeleton graph of polytopes in n dimension always have diameter polynomial in n? Linear in n? Current best known bound is $n^{O(\log n)}$.
   - Is there a polynomial time computable pivoting rule that ensures polynomial convergence of the simplex algorithm? Subexponential convergence?
   - Is there a polynomial time computable randomized pivoting rule that ensures polynomial convergence of the simplex algorithm? Subexponential convergence is known.
   - Are there strongly polynomial time (possibly randomized) algorithms for solving linear programs?

2. Beck-Fiala theorem. Improve the bounds of the discrepancy of 2-coloring hypergraphs of maximum degree d. Get below $2d - 3$ (for large d), hopefully to $O(\sqrt{d})$.

3. The following two questions are related (through the unique games conjecture):

   - Is there a polynomial time algorithm that approximates minimum vertex cover within a ratio better than $2 - \epsilon$, for some fixed $\epsilon > 0$?
   - Is there a polynomial time algorithm that approximates max-cut within a ratio better than $\alpha_{GW} \simeq 0.87856$?

4. Is there a polynomial time algorithm that colors 3-colorable graphs of maximum degree Δ with less than $\Delta^{1/3}$ colors?

5. Determine the integrality gaps for known LP relaxations (e.g., the Held-Karp LP) for Travelling Sales Person. For symmetric case, known to be between 4/3 and 3/2, and for Asymmetric case known to be between $\log n$ and 2.

There are many other less difficult research projects in these areas.

Final exam, June 12, 2pm-5pm, Zyskind 1.