Homework #1
Due: 3/11/20

1. (Strogatz 2.4) Use linearization to classify the fixed points of the following systems. Use graphical approach to explain your result, and use this approach when linearized stability fails:

\[
\begin{align*}
\dot{x} &= ax - x^3 \text{ for all possible values of } a \\
\dot{x} &= x(1-x)(2-x) \\
\dot{x} &= x^2(6-x) \\
\dot{x} &= \ln x \\
\dot{x} &= 1 - e^{-x^2}
\end{align*}
\] (1)

2. Consider a Hamiltonian system:

\[
\begin{cases}
\frac{dx}{dt} = \frac{\partial H(x,p)}{\partial p} \\
\frac{dp}{dt} = -\frac{\partial H(x,p)}{\partial x}
\end{cases}
\] (6)

(a) Prove that \(\frac{dH}{dt}(x(t), p(t)) = 0\) where \((x(t), p(t))\) denotes a solution to (6).

(b) What is the dynamical significance of the level sets of the Hamiltonian \(H(x, p) = h\)?

(c) Consider the Hamiltonian of a particle in a double well potential:

\[
H(x, p) = \frac{1}{2} p^2 - \frac{1}{2} x^2 + \frac{1}{4} x^4, \quad (x, p) \in \mathbb{R}^1 \times \mathbb{R}^1
\]

draw the level curves of \(H\) in the \((x, p)\) plane (the phase space), add time arrows, and explain the different types of motion of the particle.

(d) Think: how would the above answers change if \(H = H(x, p, t)\)?

4. List at least three (and up to 5) most important concepts learned in your reading (Chapter 1 of Meiss and/or additional material, see below).

5. List at least one result/notion which you find significant to you (related to area of your interest/difficult/non-intuitive/other).

6. Bonus I: Read chapter 3 of Meiss and solve additional ex. on pg 23-27.

7. Bonus II: find a research paper with a simple model in your field of interest (look for "toy model of .."/ "populations dynamics model of .."/ "bifurcations in ..") and describe the model: what are the dependent and independent variables, what effects where neglected and how was this justified.