

# The Third Israeli Mini-Workshop in Applied and Computational Mathematics



Organizers: Raz Kupferman, Vered Rom-Kedar, Edriss Titi


We are pleased to invite the Israeli applied math community to participate in the third Israeli Mini-Workshop in Applied and Computational Mathematics, to be held at the Hebrew University on Tuesday June 7, 2005 (the first two meetings took place on December 2003, 2004).

The idea of the workshops is to create a forum for workers in applied mathematics, especially younger faculty and students, to get to know other members of the community, and promote discussion and collaboration.

The workshop will take place in the Mathematics Department (Manchester building), Room 110, in the Givat Ram Campus of the Hebrew University. For those arriving by car, parking inside the campus will be available.

The schedule of events is as follows:

9:30-10:00	registration and opening	
10:00-10:30	David Holcman (Weizmann)	TBA
10:30-11:00	Yannis Kevrekidis (Princeton University)	Equation-free modeling of complex, multiscale systems
11:00-11:15	coffee break	
11:15-11:45	Yitschak Rubinstein (Ben-Gurion University)	Electroconvective versus electroosmotic instability in ionic solution
11:45-12:15	Eran Sharon (Hebrew University)	Shaping Thin Sheets and the Geometry of Wavy Leaves
12:15-14:00	lunch and discussions	
14:00-14:30	Leonid Prigozhin (Ben-Gurion University)	Sandpiles and superconductors: Dual variational formulations for critical-state problems
14:30-15:00	Haggai Katriel (Hebrew University)	Existence of travelling waves in systems with periodic nonlinearities: a topological method

15:00-15:15	coffee break	
15:15-15:45	Amy Novick-Cohen (The Technion)	Phase field equations with memory
15:45-16:15	Adi Ditkowski (Tel-Aviv University)	On properties of hyperbolic equations and non-reflective boundary conditions

## Registration

Participation in the workshop is free, but you are asked to register by sending an email to Raz Kupferman, so we can be adequately prepared for the day.

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## TBA

### David Holcman (Weizmann)

TBA

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## Equation-free modeling of complex, multiscale systems

### Yannis Kevrekidis (Princeton University)

Over the last five years we have been developing, with several collaborators, a systematic approach to computational coarse-graining of problems described by fine scale models (atomistic, stochastic, agent-based). This equation-free approach links continuum numerical analysis with fine scale simulation through the design of computational experiments. Theoretical and computational features of the approach will be outlined, and several applications will be discussed.

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## Electroconvective versus electroosmotic instability in ionic solution

### Yitschak Rubinstein (Ben-Gurion University)

This talk concerns the comparison of electro-convective instability in concentration polarization at an ion-selective membrane with previously reported non-equilibrium electro-osmotic instability. Electro-osmotic formulation represents an asymptotic limit case of the electro-convective one. An improved non-equilibrium electro-osmotic slip formula is presented along with the results of linear stability analysis for various non-equilibrium electro-osmotic formulations. The obtained results are compared with those for a full electro-convective formulation. It is observed that the short wave singularity typical for the non-equilibrium electro-osmotic instability is removed in the full electro-convective formulation. Effect of ionic diffusivities ratio on stability is discussed.

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# Shaping Thin Sheets and the Geometry of Wavy Leaves

**Eran Sharon (Racah Institute of Physics, Hebrew University)**

Gauss's famous theorem (theorema egregium) establishes the connection between intrinsic metric properties of a surface and its possible shapes in space. This link provides a powerful mechanism, not yet studied, for the generation of complex three dimensional shapes from thin elastic sheets, by prescribing curved metrics on them. For example the edge of a torn plastic sheet is composed of an organized cascade of waves. The waves are similar in shape but differ greatly in scale, leading to the formation of a fractal edge as an equilibrium configuration. We show that the tearing process prescribes a hyperbolic equilibrium metric near the edge of the sheet. This metric should be satisfied in order to reduce the stretching energy, but the limitations on the embedding of such a metric in Euclidean space "force" the sheet to wrinkle.

We use environmentally responsive gels to form "engineered sheets" – sheets that adopt a prescribed equilibrium metric upon induction by environmental conditions. With this system we can study the shaping mechanism in a large variety of metrics.

We suggest that some complex shapes of leaves and flowers might result from this buckling instability that links between simple growth and complex configuration. The complexity, in this case, results from elasticity and not from complex growth processes, as commonly accepted.

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## **Sandpiles and superconductors: Dual variational formulations for critical-state problems**

**Leonid Prigozhin (Blaustein Institute for Desert Research, Ben-Gurion University)**

Similar evolutionary variational inequalities appear as convenient formulations for continuous models for sandpile growth, magnetization of type-II superconductors, and evolution of some other dissipative systems characterized by the multiplicity of metastable states, long-range interactions, avalanches, and hysteresis. The origin of this similarity is that these are quasistationary models of equilibrium in which the multiplicity of metastable states is a consequence of a unilateral equilibrium condition (critical-state constraint).

Existing variational formulations for critical-state models of sandpiles and superconductors are convenient for modelling only the "primary" variables (evolving pile shape and magnetic field, respectively). The conjugate variables (the surface sand flux and the electric field) are also of interest in various applications. We present new dual variational formulations, resembling mixed variational inequalities in plasticity, for the sandpile and superconductor models. These formulations are used in numerical simulations and allow us to approximate simultaneously both the primary and dual variables.

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## **Existence of travelling waves in systems with periodic nonlinearities: a topological method**

**Haggai Katriel (Hebrew University)**

A method for proving the existence of travelling waves in some models such as discrete sine-Gordon equations and related partial differential equations, based on Leray-Schauder theory, will be presented. The

relevance of these results to the dynamics of these models, and some unresolved questions, will be discussed.

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## **Phase field equations with memory**

**Amy Novick-Cohen (The Technion)**

We explore a set of integro-differential equations known as the phase field equations with memory (Rotstein, et. al. 2001), which were constructed to describe phase transitions in the presence of "memory" i.e. slowly relaxing internal variables. After outlining their structure and various physical contexts in which these equations should be relevant, the basic features of these equations with regard to existence, uniqueness, regularity, and stability will be discussed and compared with the analogous results for similarly structured classical parabolic phase field systems. Finally, some recent qualitative predictions for the long time behaviour will be described, based in part on results of Krush-Bram, 2005.

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## **On properties of hyperbolic equations and non-reflective boundary conditions**

**Adi Ditkowski (Tel-Aviv University)**

Many hyperbolic systems, such as the Maxwell's and Acoustics equation, have plane wave solutions. If we define a domain  $\Omega \subset \mathbb{R}^m, m \geq 2$ , with smooth boundary,  $\partial \Omega$ , then at a point,  $x_0 \in \partial \Omega$ , we can classify the waves as incoming, outgoing and stationary or tangential waves. In this work we show that for systems which couple the outgoing and incoming waves, as the examples above, the incoming and outgoing waves are not separate subspaces. This implies that an incoming wave may be presented as a linear combination of outgoing waves. We use this phenomenon to prove, that for such systems, an absolutely absorbing, local and linear boundary condition, does not exist.

Joint work with Michael Sever

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