Parametric Approximation of NN

The empirical function $NN(dist, |grad|)$, which is visualized in Fig.1(c) in the paper, as well as here in Fig. B(a) (on the right), is calculated using Eq.1 from the paper:

$$NN(dist, |grad|) = Density(dist, |grad|) \cdot area(\mathcal{N}_{dist}) \ . \tag{1}$$

Given the empirical function NN, we can sample its level-sets for different fixed $NN = NN_0$. The resulting curves for $NN_0 = 9, 25, 64, 225, 625, 1500$ are displayed in Fig.B(b) using asterisks (*). For each of these level-sets the following approximation holds (Eq.2 in the paper):

$$dist(|grad|, NN) = b_1(NN) + b_2(NN) \cdot \exp(|grad|/10) \ , \tag{2}$$

where b_1 and b_2 are second order polynomials in $\sqrt{NN}$:

$$\begin{aligned} b_1(NN) &= 5 \cdot 10^{-3} NN + 0.09\sqrt{NN} - 0.044 \\ b_2(NN) &= 7.3 \cdot 10^{-4} NN + 0.3235\sqrt{NN} - 0.35. \end{aligned}$$

Fig.B(b) displays these parametric approximations. One can see that the parametric curves induced by Eq. 2 (solid lines in Fig. B(b)) approximate well the empirical samples (marked by asterisks *).

Eq. 2 is quadratic in $\sqrt{NN}$. Solving for its single valid root yields a closed-form parametric expression of NN as a function of $dist$ and $|grad|$ (Eq.3 in the paper):

$$NN(dist, |grad|) = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)^2 \ , \tag{3}$$

where

$$\begin{aligned} a &= 0.001 \cdot (5 + 0.73 \cdot \exp(|grad|/10)) \\ b &= 0.1 \cdot (0.9 + 3.24 \cdot \exp(|grad|/10)) \\ c &= -0.1 \cdot (0.44 + 3.5 \cdot \exp(|grad|/10) + dist) \ . \end{aligned}$$

The parametric approximation of NN (Eq.3) is displayed in Fig. B(c). The empirical NN and parametric NN visually appear very similar. Indeed, the average error between the two functions is 4%:

$$mean\left(\frac{|NN_{Empirical} - NN_{Parametric}|}{NN_{Empirical}}\right) = 4 \ .$$

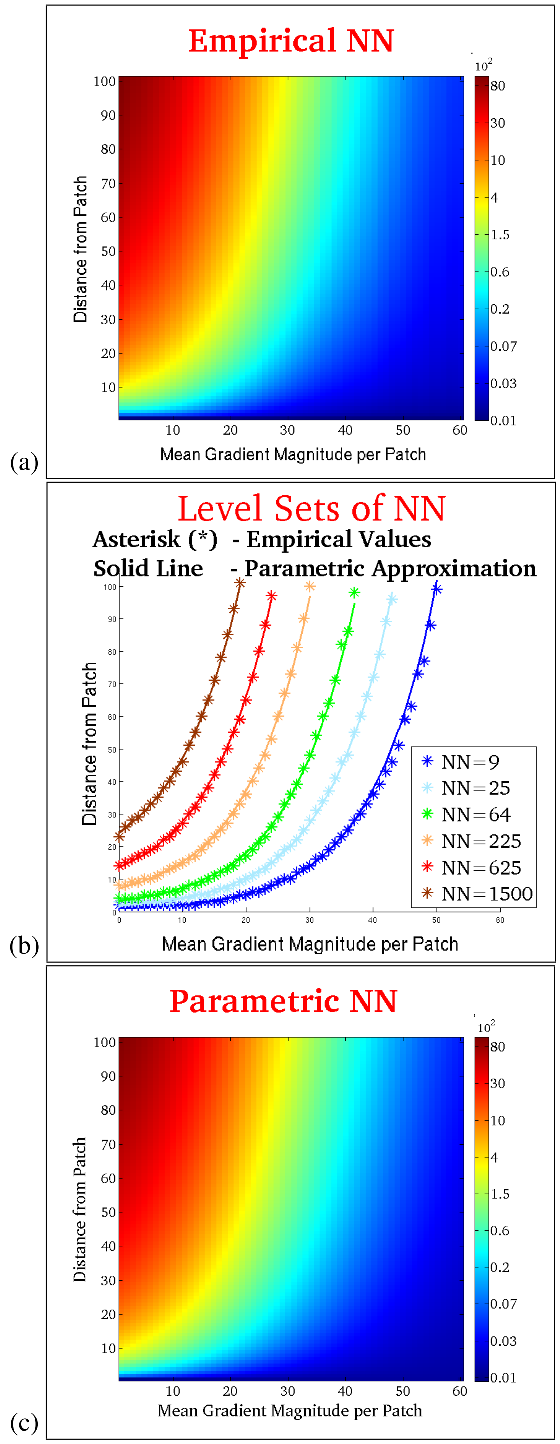

Figure B. **Parametrical Approximation of NN**.