LECTURES ON ANALYTIC DIFFERENTIAL EQUATIONS

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About the book: from the Publisher

The book combines the features of a graduate-level textbook with those of a research monograph and survey of the recent results on analysis and geometry of differential equations in the real and complex domain. As a graduate textbook, it includes self-contained, sometimes considerably simplified demonstrations of several fundamental results, which previously appeared only in journal publications (desingularization of planar analytic vector fields, existence of analytic separatrices, positive and negative results on the Riemann-Hilbert problem, Ecalle-Voronin and Martinet-Ramis moduli, solution of the Poincaré problem on the degree of an algebraic separatrix, etc.). As a research monograph, it explores in a systematic way the algebraic decidability of local classification problems, rigidity of holomorphic foliations, etc. Each section ends with a collection of problems, partly intended to help the reader to gain understanding and experience with the material, partly drafting demonstrations of the more recent results surveyed in the text.

The exposition of the book is mostly geometric, though the algebraic side of the constructions is also prominently featured. On several occasions the reader is introduced to adjacent areas, such as intersection theory for divisors on the projective plane or geometric theory of holomorphic vector bundles with meromorphic connections. The book provides the reader with the principal tools of the modern theory of analytic differential equations and intends to serve as a standard source for references in this area.

Readership

Graduate students and research mathematicians interested in analysis and geometry of differential equations in real and complex domain.

Ordering information: Visit the AMS online bookstore

Draft version is available online

The draft posted on this site is **really outdated**, with many inaccuracies, sloppy phrases and even errors. Besides, a number of sections had been added or modified. It sole purpose is to give an idea what appears in the printed book.

Do not quote this draft to avoid misleading enumeration of pages, theorems and definitions.

Contents

Preface

The branch of mathematics which deals with ordinary differential equations can be roughly divided into two large parts, *qualitative theory of differential equations* and the *dynamical systems theory.* The former mostly deals with systems of differential equations on the plane, the latter concerns multidimensional systems (diffeomorphisms on two-dimensional manifolds and flows in dimension greater than two and up to infinity). The former can be considered as a relatively orderly world, while the latter is the realm of chaos.

A key problem, in some sense a paradigm influencing the development of dynamical systems theory from its origins, is the problem of turbulence: how a deterministic nature of a dynamical system can be compatible with its apparently chaotic behavior. This problem was studied by the precursors and founding fathers of the dynamical systems theory: L. Landau, H. Hopf, A. Kolmogorov, V. Arnold, S. Smale, D. Ruelle and F. Takens. Currently this is one of the principal challenges on the crossroad between mathematics, physics and computer science. Dynamical systems theory heavily uses methods and tools from topology, differential geometry, probability, functional analysis and other branches of mathematics.

The qualitative theory of differential equations is mostly associated with autonomous systems on the plane and closely related to analytic theory of ordinary differential equations. The principal theme is investigation of local and global topological properties of phase portraits on the plane. One of the main problems of the whole area is Hilbert's sixteenth problem, the question on the number and position of limit cycles of a polynomial vector field on the plane. In a very broad sense this can be assessed as the question: to which extent properties of polynomials defining a differential equation are inherited by its absolutely transcendental (and sometimes very weird) solutions.

Another major part of analytic theory of differential equations is the linear theory. Here the key problem is Hilbert's twenty-first problem, also known as the Riemann–Hilbert problem, which has a long dramatic history and was solved "only yesterday". Discussion of this problem constitutes an important part of this book.

The qualitative theory of differential equations was essentially created in the works by H. Poincaré who discovered that differential equations belong not only to the realm of analysis, but also to geometry. Deriving geometric properties of solutions directly from the equations defining them, was his principal idea. These ideas were further developed in each of the two branches separately, but their present appearance looks very different.

Differential equations brought into existence such areas of mathematics as topology and Lie groups theory. In turn, the analytic theory of differential equations is not a closed area, but rather provides a source of applications and motivation for other disciplines. In this book we stress using complex analysis, algebraic geometry and topology of vector bundles, with some other interesting links briefly outlined at the appropriate places.

On the frontier between differential equations and the singularity theory, lies the notion of a normal form, one of the central concepts of this book. The first chapter contains the basics of formal and analytic normal form theory. The tools developed in this chapter are systematically used throughout the book. The study of phase portraits of composite singular points requires elaboration of the blowing-up technique, another classical tool known for over a century. The famous Bendixson desingularization theorem is proved in our textbook by transparent methods.

A new approach to local problems of analysis, based on the notion of algebraic and analytic solvability, was suggested by V. Arnold and R. Thom around forty years ago. In Chapter II we treat from this point of view the local theory of singular points of planar vector fields. It is proved that the stability problem and the problem of topological classification of planar vector fields are algebraically solvable in all cases except for the center/focus dichotomy. This dichotomy is algebraically unsolvable, as is proved in the same chapter. Besides these topics, the chapter contains local analysis of singular points of holomorphic foliations: the proof of the C. Camacho–P. Sad theorem on existence of analytic separatrices through singular points, integrability via the local holonomy group as discovered by J.-F. Mattei and R. Moussu, and demonstration of the Bautin theorem on small limit cycles of quadratic vector fields.

The third chapter deals with the linear theory. Somewhat paradoxically, application of normal forms of nonlinear systems to investigation of linear systems considerably simplifies exposition of many classical results. The chapter contains a succinct derivation of some positive and negative results on solvability of the Riemann–Hilbert problem.

Chapter IV deals with a new direction in the theory of normal forms, the functional moduli of analytic classification of resonant singularities. The main working tool used in this study is an almost complex structure and quasiconformal maps. The latter already played a revolutionizing role in the nearby theory of holomorphic dynamics. The main basic facts from these theories are briefly summarized in this chapter. The chapter ends with the proof of the "easy version" of the finiteness theorem for limit cycles of analytic vector fields, with an additional assumption that all singular points are hyperbolic saddles. The proof illustrates the power of using local normal forms in the solution of problems of a global nature.

Chapter V is concerned with the global theory of polynomial differential equations on the real and complex plane, bridging between algebraic, "almost algebraic" and essentially transcendental questions.

The chapter begins with the solution of the Poincaré problem on the maximal degree which can have an algebraic solution of a polynomial differential equation (a relatively recent spectacular result due to D. Cerveau, A. Lins Neto and M. Carnicer). The second section focuses on the interaction between the theory of Riemann surfaces and global theory of differential equations. We describe the topology of stratification of the complex projective plane by level curves of a generic bivariate polynomial, including derivation of the Picard–Lefschetz formulas for the Gauss–Manin connexion. This is the main working tool for deriving certain inequalities for the number of zeros of complete Abelian integrals, a question very closely related to Hilbert's sixteenth problem. Finally, we discuss generic properties of complex foliations that are very often drastically different from their real counterparts. For instance, finiteness of limit cycles on the real plane is in sharp contrast with a typically infinite number of the complex limit cycles, and the structural stability of real phase portraits counters rigidity of a generic complex foliation.

Some basic facts from complex analysis in several variables frequently used in the book, are recalled in the Appendix.

Almost all sections are ended by the problem lists. Together with easy problems, sometimes called exercises, the lists contain difficult ones, lying on the frontier of the current research.

The book was not intended to serve as a comprehensive treatise on the whole analytic theory of ordinary differential equations. The selection of topics was based on the personal taste of the authors and restricted by the size of the book. We do not even mention such classical areas as the theory of Riccati and Painlevé equations, the Malmquist theorem, integral representations and transformations. We omit completely the differential Galois theory, resurgent functions introduced by Ecalle and the fewnomial theory invented by A. Khovansky. Nevertheless, the subjects covered in the book constitute in our opinion a connected whole revolving around few key problems that play an organizing role in the development of the entire area.

Exposition of each topic begins with basic definitions and reaches the present-day level of research on many occasions. Traditionally, the proofs of many results of analytic theory of differential equations are very technically involved. Whenever available, we tried to preface formulas by motivations and avoid as much as possible all cumbersome and nonrevealing computations.

The book is primarily aimed at graduate students and professionals looking for a quick and gentle initiation into this subject. Yet experts in the area will find here several results whose complete exposition was never published before in books. On the other hand, undergraduate students should be able to read at least some parts of the book and get introduced into the beautiful area that occupies a central position in modern mathematics.

* * *

The idea to write this book, especially the chapter on linear systems, was to a large extent inspired by the recent dramatic achievements by our dear friend and colleague **Andrei Bolibruch**, who solved one of the most challenging problems of analytic theory of ordinary differential equations, the Riemann-Hilbert problem. Andrei read several first drafts of this chapter and his comments and remarks were extremely helpful.

On November 11, 2003, at the age 53, after a long and difficult struggle, Andrei Andreevich Bolibruch succumbed to the grave disease. This book is a posthumous tribute to his mathematical talents, artistic vision and impeccable taste with which he always chose problems and solved them.

* * *

When the work on this book (which took a much longer time than initially expected) was essentially over, another similar treatise appeared. In 2006 Henryk Żołądek published the fundamental monograph [**Żoł06**] titled very tellingly "The Monodromy Group". The scope of both books is surprisingly similar, though the symmetric difference is also very large. Yet most of the subjects which simultaneously occur in the two books are treated in rather different ways. This gives a reader a rare opportunity to choose the exposition that is closer to his/her heart: the mathematics can be the same but our ways of speaking about it differ.

* * *

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